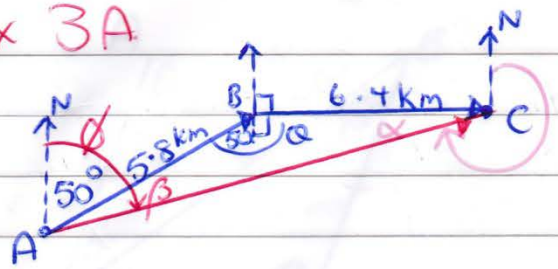
Specialist Mathematics Unit 1: **Chapter 3**

Ex 3A

1.



$\theta = 50 + 90 = 140°$

$AC^2 = (5.8)^2 + (6.4)^2 - 2(5.8)(6.4)\cos 140$

$AC = 11.466\text{km} \approx 11.5\text{km}$

$\frac{6.4}{\sin\beta} = \frac{11.466}{\sin 140}$

$\beta = 21.025°$

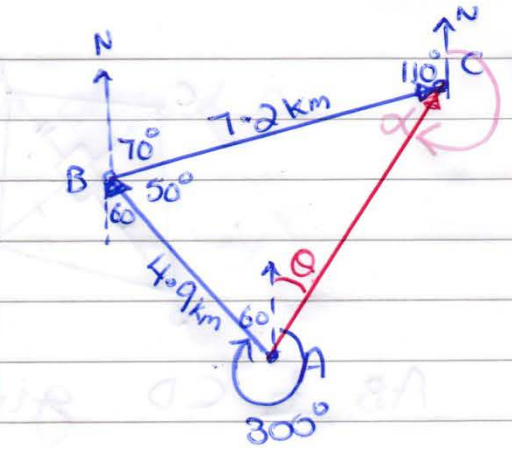
∴ bearing of C from A
$= 50 + \beta$
$= 50 + 21.025$
$= 71.025° \approx 071°$

$\alpha = 180 - 140 - 21.025$
$\alpha = 18.9745°$

∴ Bearing of A from C

$= 270 - 18.9745°$
$= 251.0255$
$\approx 251°$

2.



$AC^2 = 4.9^2 + 7.2^2 - 2(4.9)(7.2)\cos 50$

$AC = 5.522\text{km} \approx 5.5\text{km}$

$\frac{7.2}{\sin(\theta+60)} = \frac{5.522}{\sin 50}$

$\theta = 27.177$

Bearing of C from A
$= \theta = 027°$

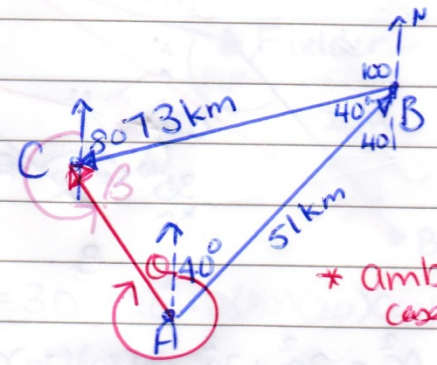
$\alpha = 180 - 50 - 60 - 27.177$
$\alpha = 42.823°$

∴ Bearing of A from C

$360 - (110 + 42.823)$
$= 207.177$

$\approx 207°$

3.



$AC^2 = 73^2 + 51^2 - 2(73)(51)\cos 40°$

$AC = 47.18$ km

≈ 47 km

Bearing of C from A

$\frac{73}{\sin(\theta + 40)} = \frac{47.18}{\sin 40}$

* largest side is opp largest angle ∴ CAB is obtuse.

$\theta = 55.987$

∴ Bearing $= 360 - 55.987°$

$= 315.987$

$\approx 304.013°$

∴ 304°

Bearing of A from C.

$\frac{51}{\sin\beta} = \frac{73}{\sin 95.989}$ * use obtuse angle to ensure △ sum is 180

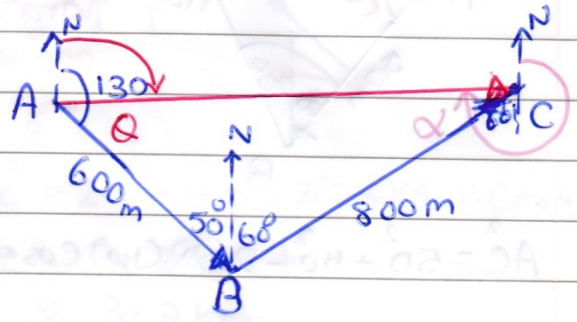
$\beta = 44.0126°$

∴ Bearing $= 80 + 44.0126$

$= 124.0126°$

$\approx 124°$

4. * I have used trigonometry instead of scale drawings!



$AC^2 = 600^2 + 800^2 - 2(600)(800)\cos 110$

$AC = 1152.536$ km

Bearing of C from A.

$\frac{800}{\sin\theta} = \frac{1152.536}{\sin 110}$

$\theta = 40.712°$

∴ Bearing is $130 - 40.712$

$= 89.2877$

$\approx 089°$

Bearing of A from C

$\frac{600}{\sin\alpha} = \frac{1152.536}{\sin 110}$

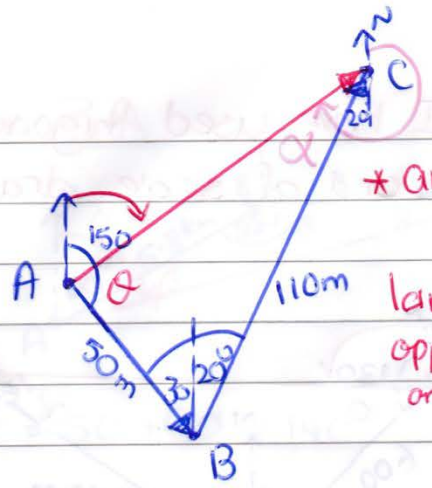
$\alpha = 29.2877°$

∴ Bearing is $180 + 60 + 29.2877$

$= 269.2877$

$\approx 269°$

5.



* ambigious case

largest side opp larger angle

$AC^2 = 50^2 + 110^2 - 2(50)(110)\cos 50$

$AC = 86.7717$ km

≈ 87 km

Bearing of C from A

$\frac{110}{\sin\theta} = \frac{86.7717}{\sin 50}$

$\theta = 76.194^\circ$ (103.805°)

ambigious case?

$\therefore$ Bearing is $150 - 103.805$

$= 46.19^\circ$

$\approx 046^\circ$

Bearing of A from C

$\frac{50}{\sin\alpha} = \frac{110}{\sin 103.805}$

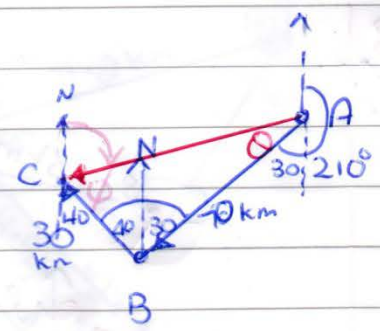
$\alpha = 26.19^\circ$

$\therefore$ Bearing is $180 + 20 + 26.19$

$= 226.194$

$\approx 226^\circ$

6.



$AC^2 = 30^2 + 70^2 - 2(30)(70)\cos 70^\circ$

$AC = 66.05691$ km

≈ 66 km

Bearing of C from A

$\frac{30}{\sin\theta} = \frac{66.05691}{\sin 70}$

$\theta = 25.262^\circ$

$\therefore$ Bearing $= 210 + 25.262$

$= 235.26^\circ$

$\approx 235^\circ$

Bearing of A from C

$\phi = 180 - 70 - 25.262^\circ$

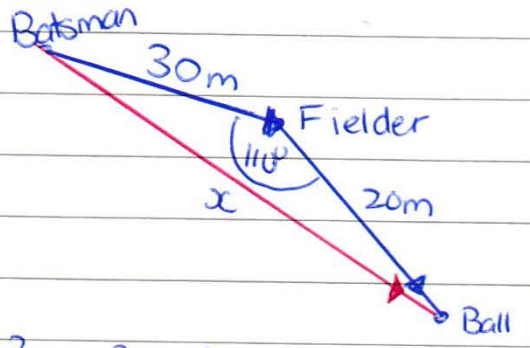
$\phi = 84.738^\circ$

$\therefore$ Bearing $= 180 - (40 + 84.738)$

$= 55.26$

$\approx 055^\circ$

7.

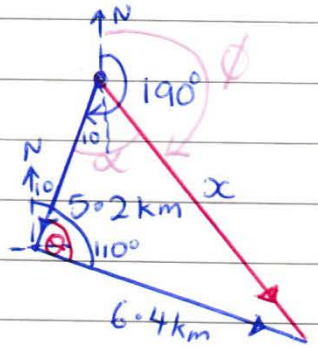


$x^2 = 30^2 + 20^2 - 2(30)(20)\cos 110$

$x = 41.357$

$\approx 41\text{m}$

8



$\theta = 110 - 10 = 100^2$

$x^2 = 5.2^2 + 6.4^2 - 2(5.2)(6.4)\cos 100$

$x = 8.919$

$\approx 8.9\text{ km}$

$\frac{6.4}{\sin\alpha} = \frac{8.919}{\sin 100}$

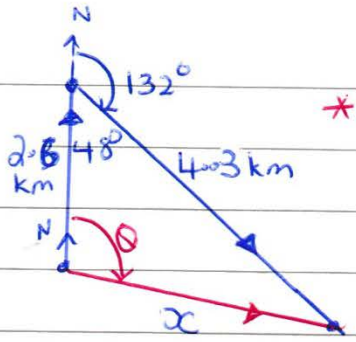
$\alpha = 44.96^\circ$

$\therefore$ Bearing $= 190^\circ - 44.96^\circ$

$\phi = 145.04$

$\approx 145^\circ$

9.



*ambigious case

$x^2 = 2.6^2 + 4.3^2 - 2(2.6)(4.3)\cos 48^\circ$

$x = 3.2075\text{ km}$

$\approx 3.2\text{ km}$

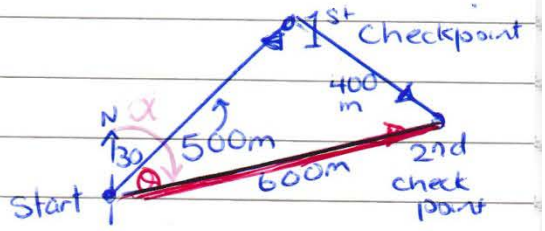
Bearing: $\frac{4.3}{\sin\theta} = \frac{3.2075}{\sin 48}$

$\theta = 94.958$

$\theta \approx 95^\circ$

$\therefore$ bearing $= 095^\circ$

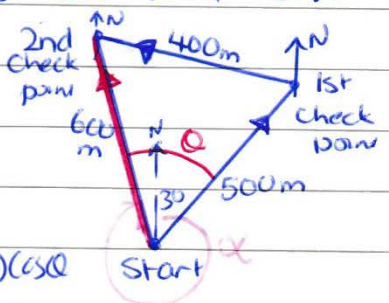
10.



$400^2 = 500^2 + 600^2 - 2(500)(600)\cos\theta$

$\theta = 41.409$

$\therefore$ Bearing is $30 + 41.4 = 071^\circ$



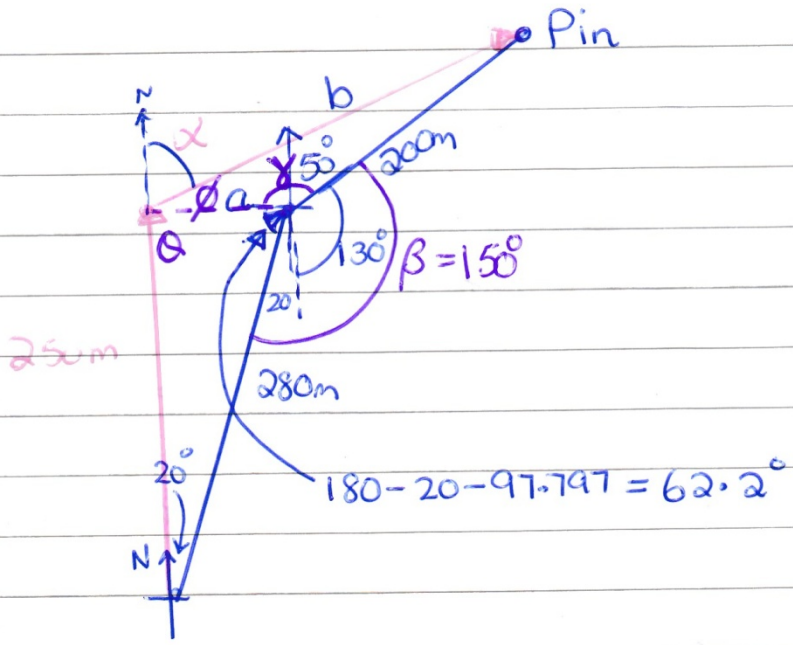
$400^2 = 500^2 + 600^2 - 2(500)(600)\cos\theta$

$\theta = 41.409$

$\therefore$ Bearing $\Rightarrow$ $41.409 - 30 = 11.4096^\circ$

$\therefore \alpha = 360 - 11.4096^\circ = 349^\circ$

11.



$a^2 = 250^2 + 280^2 - 2(250)(280)\cos 20$

$a = 96.6594\text{m}.$

$\frac{280}{\sin\theta} = \frac{96.6594}{\sin 20}$

$\theta = 97.797°$

$\gamma = 360 - 150 - 62.2°$

$\gamma = 147.797°$

$b^2 = 96.6594^2 + 200^2 - 2(96.6594)(200)\cos 147.797$

$b = 286.459$

$\approx 286\text{m}$

$\frac{200}{\sin\emptyset} = \frac{286.459}{\sin 147.797}$

$\emptyset = 21.8436°$

∴ Bearing

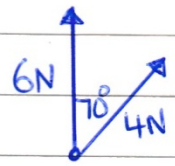
$\alpha = 180 - (21.8436 + 97.797)$

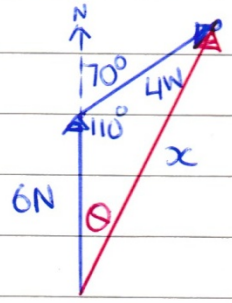
$= 60.359$

$\approx 060°$

Ex 3B.

1.





$x^2 = 6^2 + 4^2 - 2(6)(N)\cos 110^\circ$

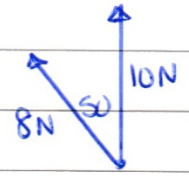
$x = 8.27N = 8.3N$

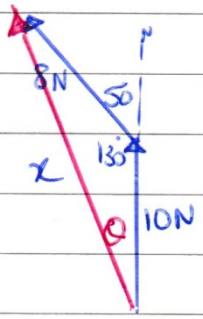
$\frac{4N}{\sin\theta} = \frac{8.27N}{\sin 110}$

$\theta = 27.028 = 027^\circ$

ie 27° to the vertical

2.





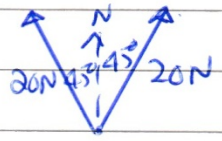
$x^2 = 8^2 + 10^2 - 2(8)(10)\cos 130^\circ$

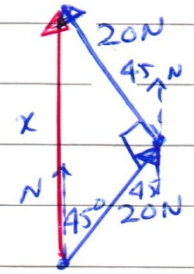
$x = 16.335 = 16.3N$

$\frac{8}{\sin\theta} = \frac{16.3}{\sin 130}$ $\quad \theta = 22.03^\circ$

$\theta = 22^\circ$ to the vertical.

3





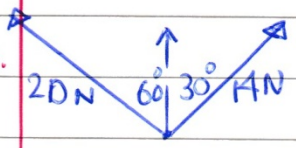
$x^2 = 45^2 + 45^2 - 2(45)(45)\cos 90$

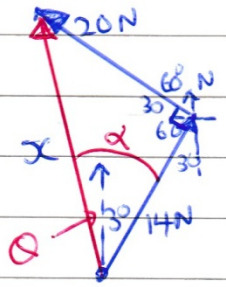
$x = 63.639$

$= 63.6N$

$\theta = 0^\circ$ to the vertical

4.





$x^2 = 20^2 + 14^2 - 2(20)(14)\cos 90$

$x = 24.413N$

$= 24.4N$

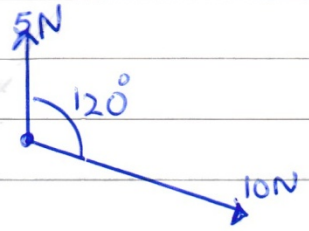
$\frac{20}{\sin\alpha} = \frac{24.413}{\sin 90}$

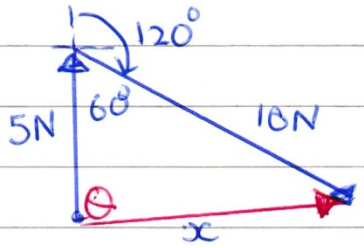
$\alpha = 55.00798^\circ$

$\therefore \theta = 55 - 30 = 25^\circ$

25° to the vertical

5.





$x^2 = 5^2 + 10^2 - 2(5)(10)\underbrace{\cos 60}_{\frac{\sqrt{3}}{2}}$

$x = 8.66\ N$

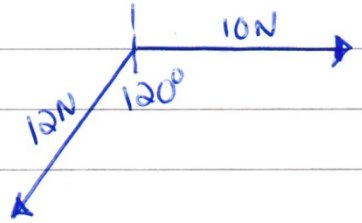
$x = 5\sqrt{3}\ N$

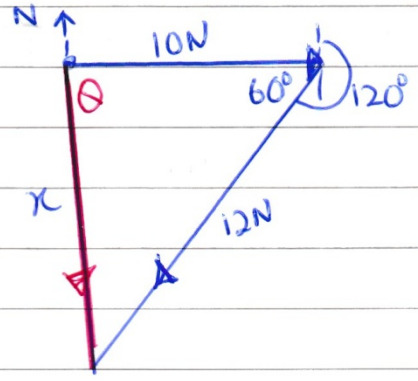
$\frac{10}{\sin\theta} = \frac{5\sqrt{3}}{\sin 60}$

$\theta = 90°$

$\therefore$ bearing $= 090°$

6.





$x^2 = 12^2 + 10^2 - 2(12)(10)\underbrace{\cos 60}_{\frac{\sqrt{3}}{2}}$

$x = 2\sqrt{31}$

$\frac{12}{\sin\theta} = \frac{2\sqrt{31}}{\sin 60}$

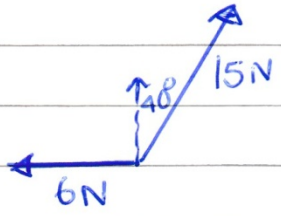
$\theta = 68.95$

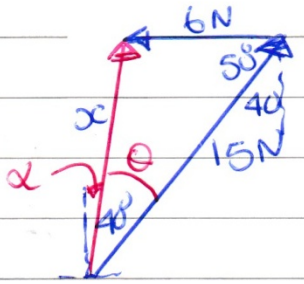
$\therefore$ bearing $= 90 + 68.95$

$= 158.95$

$\approx 159°$

7.





$x^2 = 6^2 + 15^2 - 2(6)(15)\cos 50$

$x = 12.0539$

$= 12.1\ N$

$\frac{6}{\sin\theta} = \frac{12.1}{\sin 50}$

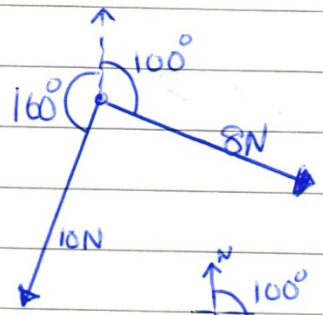
$\theta = 22.415$

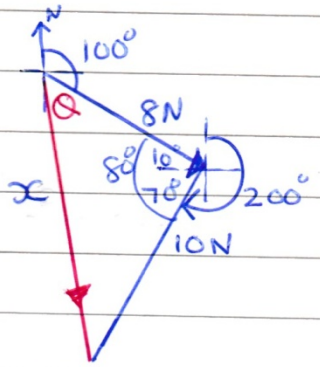
Bearing $\alpha = 40 - 22.415$

$= 17.58$

$= 018°$

8.





$x^2 = 8^2 + 10^2 - 2(8)(10)\cos 80$

$x = 11.67$

$= 11.7N$

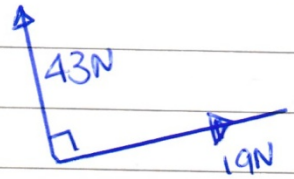
$\frac{10}{\sin\theta} = \frac{11.67}{\sin 80}$

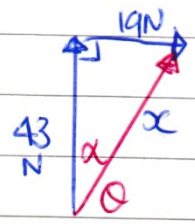
$\theta = 57.54°$

Bearing $= 100 + 57.54$

$= 157.54 \approx 158°$

9.



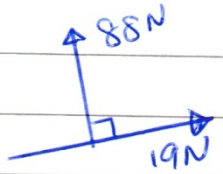


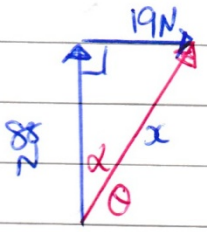
$x^2 = 43^2 + 19^2$ $x = 47.01 = 47N$

$\tan\alpha = \frac{19}{43}$ $\alpha = 23.83$

$\therefore \theta = 90 - 23.83 = 66°$

10.



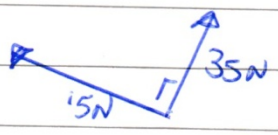


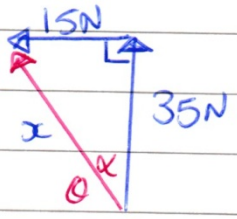
$x^2 = 88^2 + 19^2$ $x = 90.03 = 90N$

$\tan\alpha = \frac{19}{88}$ $\alpha = 12.18$

$\therefore \theta = 90 - 12.18 = 77.8 \approx 78°$

11.





$x^2 = 15^2 + 35^2$ $= 38.08 = 38N$

$\tan\alpha = \frac{15}{35}$ $\alpha = 23.199°$

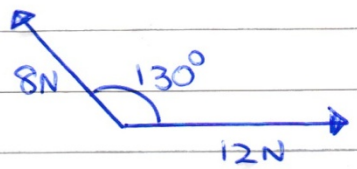
$\therefore \theta = 90 - 23.199$

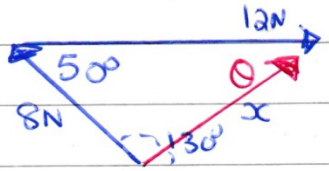
$\theta = 66.8$

$\approx 67°$

Ex 3C.

12.





$x^2 = 8^2 + 12^2 - 2(8)(12)\cos 50$

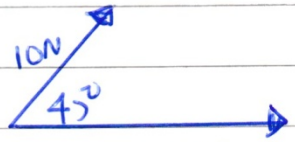
$x = 9.196998$

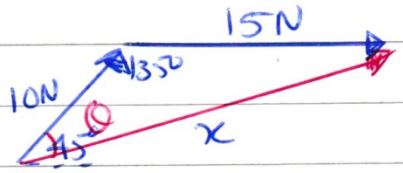
$x = 9.2\,N$

$\frac{8}{\sin\theta} = \frac{9.2}{\sin 50} \Rightarrow \theta = 41.785°$

$\theta = 42°$

13.





$x^2 = 10^2 + 15^2 - 2(10)(15)\cos 135$

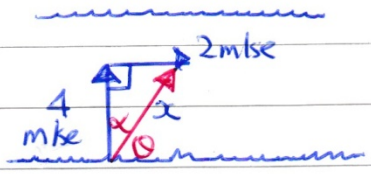
$x = 23.176$

$= 23.2\,N$

$\frac{15}{\sin\theta} = \frac{23.176}{\sin 135} \rightarrow \theta = 27.236$

$\theta = 27°$

1.



$x^2 = 4^2 + 2^2$

$x = 4.47 = 4.5\ m/sec$

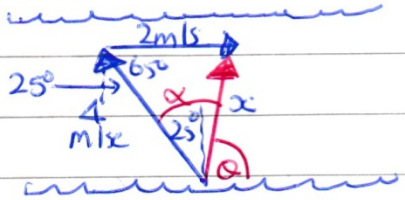
$\tan\alpha = \frac{2}{4} \quad \alpha = 26.565$

$\therefore \theta = 90 - 26.565$

$\theta = 63.435$

$= 63°$

2



$x^2 = 4^2 + 2^2 - 2(4)(2)\cos 65$

$x = 3.638$

$= 3.6\ N/sec$

$\frac{2}{\sin\alpha} = \frac{3.6}{\sin 65}$

$\alpha = 29.88°$

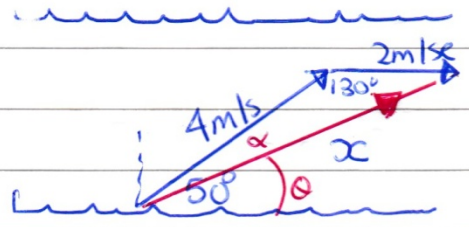
$\therefore \theta = 90 - (29.88 - 25)$

$\theta = 90 - 4.88$

$= 85.1198$

$= 85°$

3.



$x^2 = 4^2 + 2^2 - 2(4)(2)\cos 130$

$x = 5.503$

$x = 5.5 \text{ m/sec}$

$\frac{2}{\sin\alpha} = \frac{5.5}{\sin 130}$

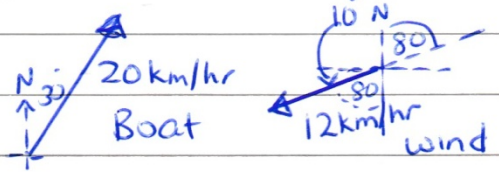
$\alpha = 16.16°$

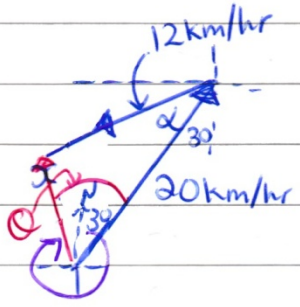
$\therefore \theta = 50 - 16.16$

$\theta = 33.83$

$\approx 34°$

4.





$\alpha = 90 - 30 - 10 = 50°$

$x^2 = 12^2 + 20^2 - 2(12)(20)\cos 50$

$x = 15.344 = 15.3 \text{ km/hr}$

$\frac{12}{\sin\theta} = \frac{15.344}{\sin 50} \rightarrow \theta = 36.8°$

$\therefore$ bearing is

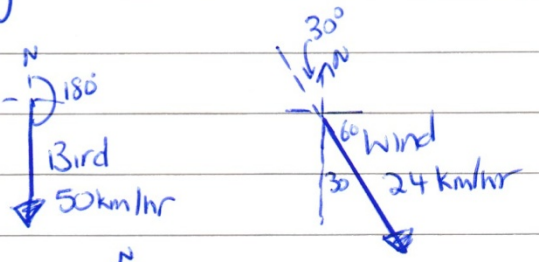
$36.8 - 30 = 6.8°$

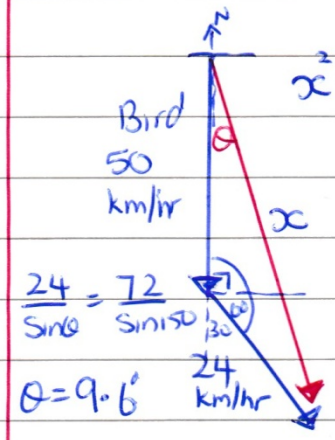
$360 - 6.8° = 353.19$

$\approx 353°$

* draw a reasonable diagram to ensure you get correct angles.

5.





$x^2 = 50^2 + 24^2 - 2(50)(24)\cos 150$

$x = 71.8 \text{ km/hr}$

$= 72 \text{ km/hr}$

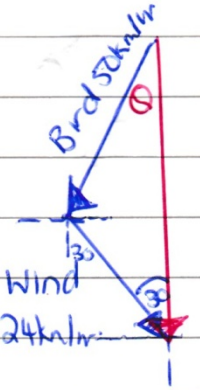
$\frac{24}{\sin\theta} = \frac{72}{\sin 150}$

$\theta = 9.6°$

$\therefore$ bearing $180 - 9.6$

$= 170°$

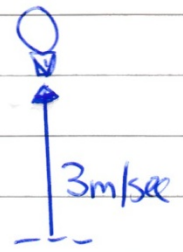
we want the resultant to be due south!



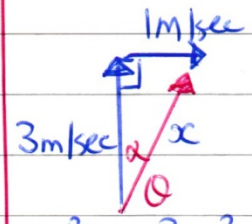
$\frac{24}{\sin\theta} = \frac{50}{\sin 30} \rightarrow \theta = 13.88$

$\therefore$ bearing is $180 + 13.88$

$= 194°$

6. 

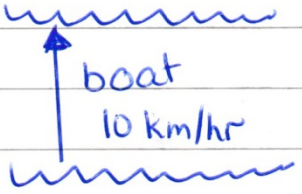
a) after 1 min
$3 \times 60 = 180m$

b) 

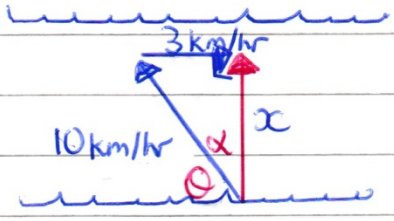
$x^2 = 3^2 + 1^2 = 3.1623$ m/sec
$\sqrt{10} = 3.2$ m/sec

c) $\tan\alpha = \frac{1}{3}$ $\alpha = 18.43$

$\therefore \theta = 90 - 18.43$
$= 71.56$
$\approx 72°$

7. 

a) current 3km/hr →



$x^2 = 10^2 - 3^2 = \sqrt{91}$ km/hr

$\sin\alpha = \frac{3}{10}$ $\alpha = 17.457$

$\therefore \theta = 72.54° \approx 73°$
upstream to current

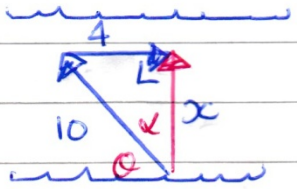
$s = \frac{d}{t}$ $t = \frac{d}{s}$

80m ⇒ $\frac{0.08\text{km}}{\sqrt{91}\text{ km/hr}}$

$= 0.008386$ hr
$= \times 60 \times 60$
min sec

$= 30.19$ sec
≈ 30 sec

b) current 4km/hr



$x^2 = 10^2 - 4^2 = \sqrt{84} = 9.17$ km/hr

$\sin\alpha = \frac{4}{10}$ $\alpha = 23.578$

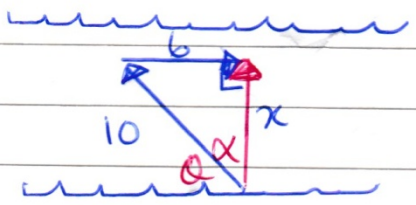
$\therefore \theta = 90 - 23.578 = 66.4$
upstream at $\approx 66°$

$t = \frac{0.08}{\sqrt{84}\text{ km/hr}}$

$= 0.00873$ hr
$\times 60 \times 60$
min sec

$= 31.42$
≈ 31 sec

7c) current 6km/hr →



$x^2 = 10^2 - 6^2 = 8$ km/hr

$\sin\alpha = \frac{6}{10}$ $\alpha = 36.87$

$\therefore \theta = 90 - 36.87 = 53.1$

upstream at ≈ 53°

$t = \frac{0.08}{8\text{km/hr}}$

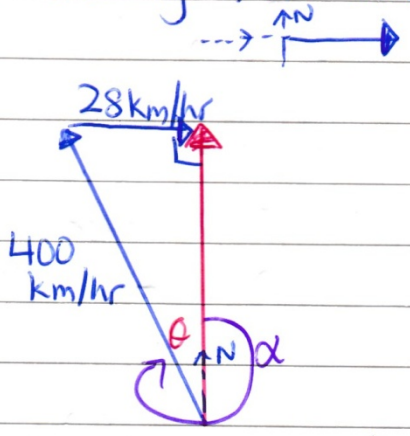
$= 0.01$ hrs

$\times 60 \times 60$ sec

$= 36$ sec

8. plane wants to travel due north

Wind blowing "from" west



$\sin\theta = \frac{28}{400}$ $\theta = 4.01°$

$\therefore$ bearing of $\alpha = 360 - 4.01$

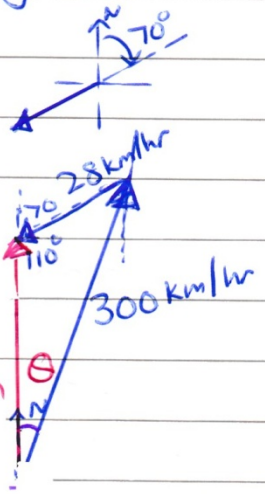
$= 355.98$

≈ 356°

9. plane wants to fly due north

Wind is blowing "from" 070°

28km/hr

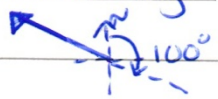


$\frac{28}{\sin\theta} = \frac{300}{\sin 110}$

$\theta = 5.03°$

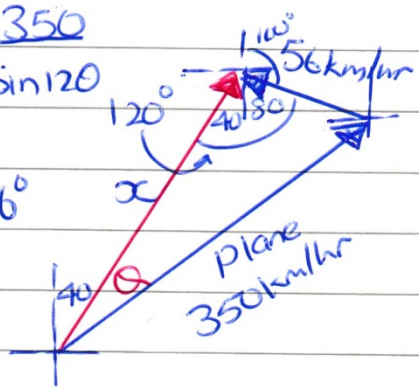
bearing = 005°

10. wind blowing "from" 100°



plane wants to fly 040°

$\frac{56}{\sin\theta} = \frac{350}{\sin 120}$



$\theta = 7.96°$

$\therefore$ fly at $40 + 7.96$

$= 47.96$

≈ 048°

$350^2 = x^2 + 56^2 - 2(x)(56)\cos 120$

$x = 318.624$ km/hr

$s = \frac{d}{t}$ $\quad t = \frac{d}{s}$

airport is 500km away

$t = \frac{500}{318.624}$

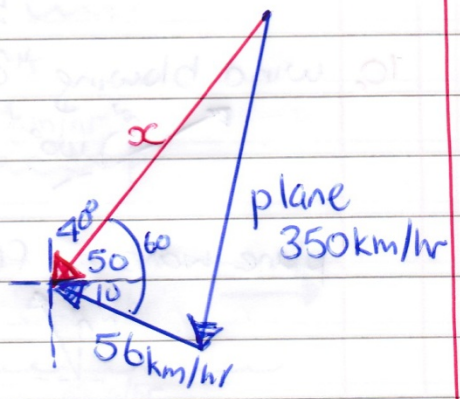
$t = 1.569$ hr

$= 1$hr $\quad 0.569 \times 60$

$= 34$ mins

$\therefore$ 1hr 34 mins

b) to come back to start



$350^2 = 56^2 + x^2 - 2(56)(x)\cos 60$

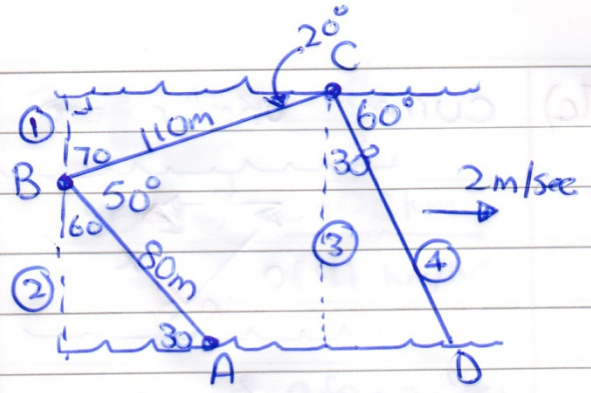
$x = 374.62$ km/hr

$t = \frac{500}{374.62} = 1.335$hrs

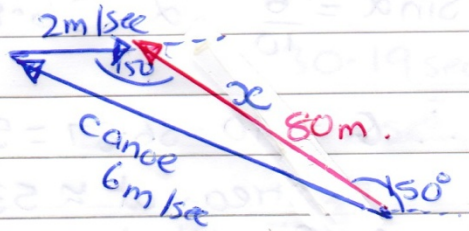
0.335×60 min

$=$ 1hr 20min $\quad$ 20min

11.



a) A to B



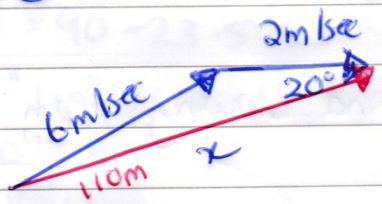
$6^2 = 2^2 + x^2 - 2(x)(2)\cos 150°$

$x = 4.184$ m/sec

$s = \frac{d}{t} \rightarrow t = \frac{d}{s} = \frac{80}{4.184}$

$= 19.12$sec.

b) B to C



$6^2 = x^2 + 2^2 - 2(x)(2)\cos 20$

$x = 7.84$ m/sec

$t = \frac{110}{7.84} = 14.03$ sec

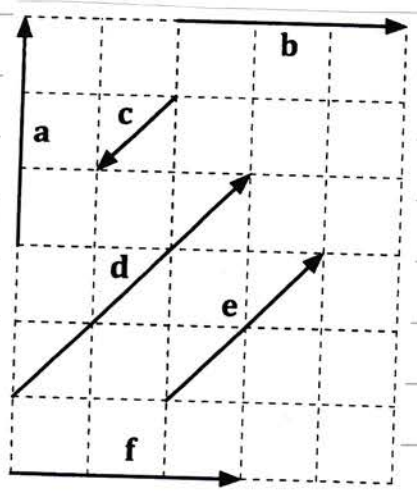
① $\Rightarrow \cos 70 = \frac{①}{110}$

$① = 110\cos 70$

$① = 37.62$m

Ex 3D.

1.



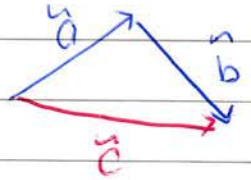
a) like & unequal
$\vec{d}$, $\vec{e}$

b) unlike parallel & unequal
$\vec{c}$ & $\vec{d}$ and $\vec{c}$ & $\vec{e}$

c) same magnitude but not equal
$\vec{a}$ & $\vec{b}$ $\vec{a}$ & $\vec{f}$

d) two equal vectors.
$\vec{b}$ & $\vec{f}$

2. $\vec{a} + \vec{b} = \vec{c}$



② ⇒ $\sin 30 = \frac{②}{80}$

$② = 80 \sin 30$
$= 40m$

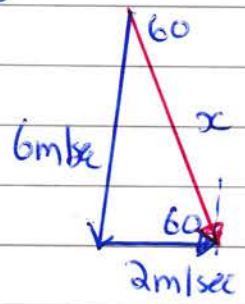
∴ ③ = ① + ②
$= 77.622\ m$

④ ⇒ $\cos 30 = \frac{③}{④}$

$④ = \frac{③}{\cos 30}$

$④ = 89.63\ m$

c) C to D



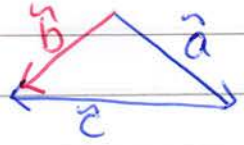
$6^2 = 2^2 + x^2 - 2(2)(x)\cos 60$
$x = 6.6745\ m/sec$

$t = \frac{89.63}{6.6745} = 13.289$
$= 13.3 sec$

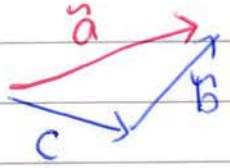
total $= 19.1 + 14.0 + 13.3$

$= 46.4$ sec
≈ 46 sec.

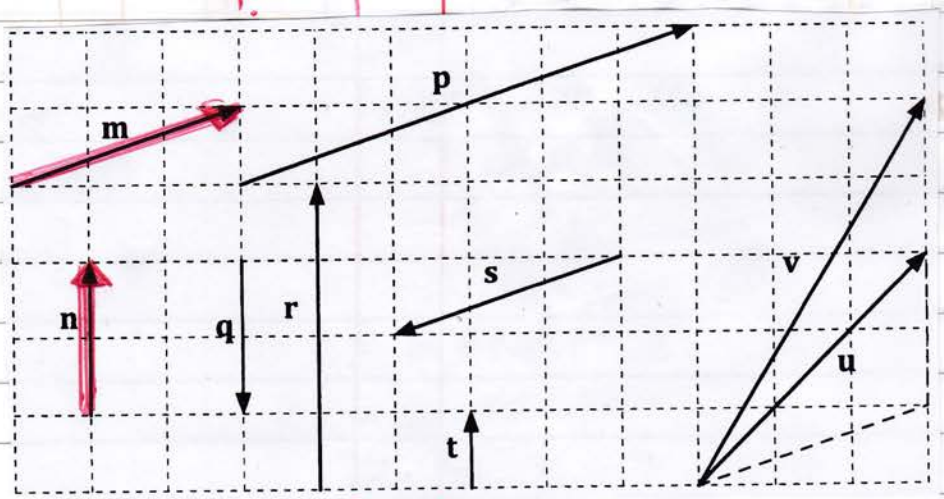
2b) $\underset{\frown}{a} + \underset{\frown}{c} = \underset{\frown}{b}$



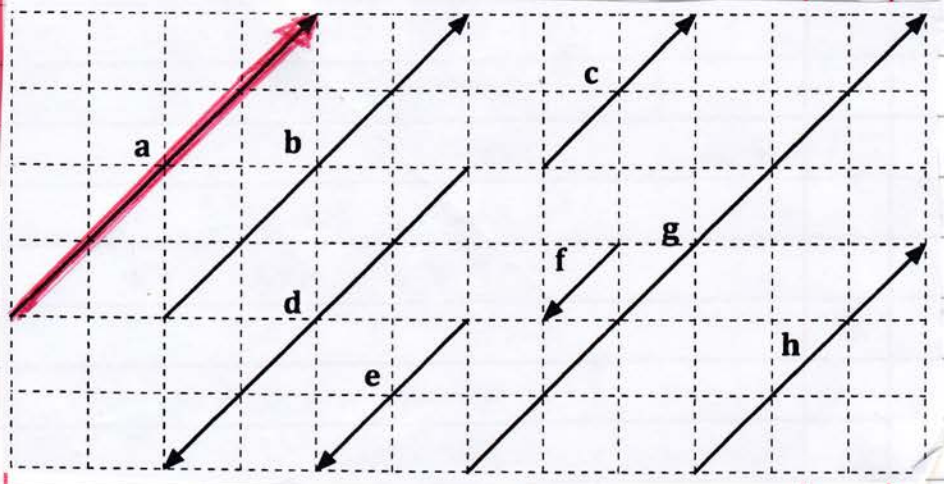
c) $\underset{\frown}{b} + \underset{\frown}{c} = \underset{\frown}{a}$



4.



3.



$\underset{\frown}{p} = 2\underset{\frown}{m}$

$\underset{\frown}{q} = -\underset{\frown}{n}$

$\underset{\frown}{r} = 2\underset{\frown}{n}$

$\underset{\frown}{s} = -\underset{\frown}{m}$

$\underset{\frown}{t} = \frac{1}{2}\underset{\frown}{n}$

$\underset{\frown}{u} = \underset{\frown}{m} + \underset{\frown}{n}$

$\underset{\frown}{v} = \underset{\frown}{m} + 2\underset{\frown}{n}$

$\underset{\frown}{b} = \underset{\frown}{a}$

$\underset{\frown}{d} = -\underset{\frown}{a}$

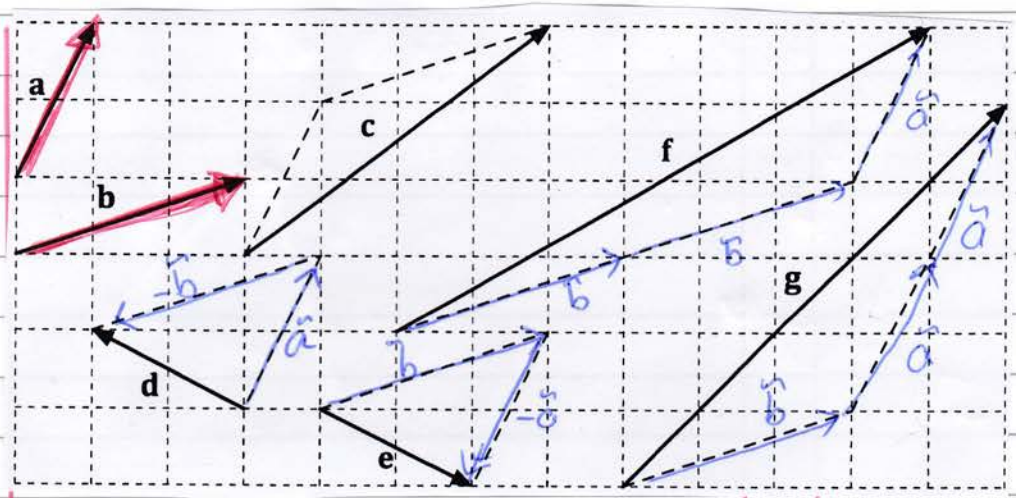
$\underset{\frown}{e} = -\frac{1}{2}\underset{\frown}{a}$

$\underset{\frown}{f} = -\frac{1}{4}\underset{\frown}{a}$

$\underset{\frown}{g} = 1\frac{1}{2}\underset{\frown}{a}$ or $\frac{3}{2}\underset{\frown}{a}$

$\underset{\frown}{h} = \frac{3}{4}\underset{\frown}{a}$

5.

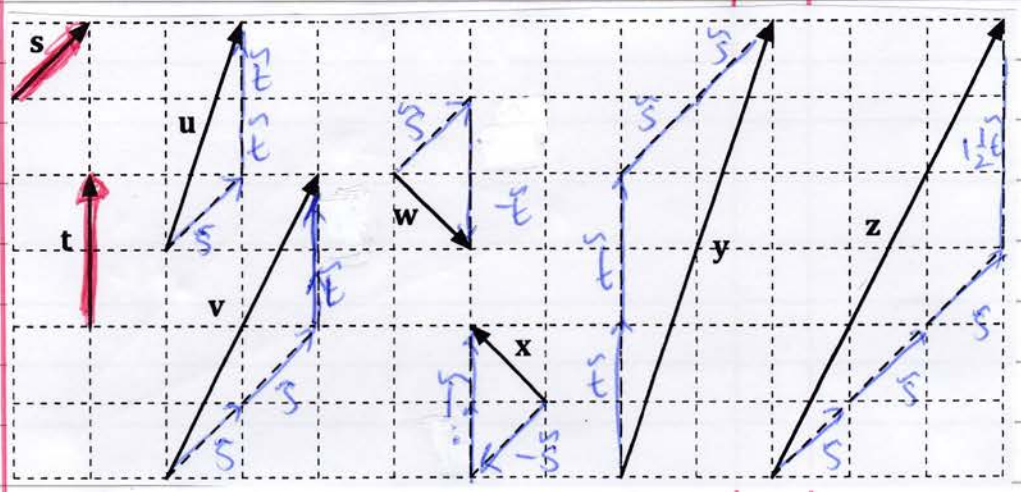


$\vec{c} = \vec{a} + \vec{b}$ $\quad \vec{d} = \vec{a} - \vec{b}$

$\vec{e} = \vec{b} - \vec{a}$ $\quad \vec{f} = 2\vec{b} + \vec{a}$

$\vec{g} = \vec{b} + 2\vec{a}$

6.



$\vec{u} = \vec{s} + 2\vec{t}$ $\quad \vec{y} = 2\vec{t} + 2\vec{s}$

$\vec{v} = 2\vec{s} + \vec{t}$ $\quad \vec{z} = 3\vec{s} + 1.5\vec{t}$

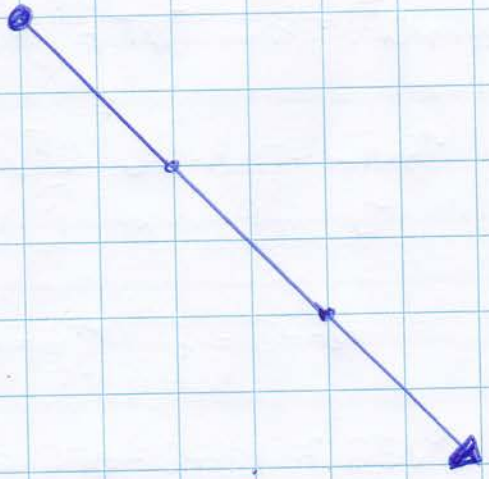
$\vec{w} = \vec{s} - \vec{t}$

$\vec{x} = -\vec{s} + \vec{t}$ or $\vec{t} - \vec{s}$

7.

a) $2\vec{a}$

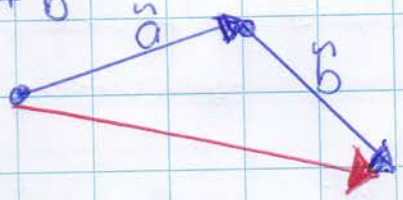
b) $3\vec{b}$



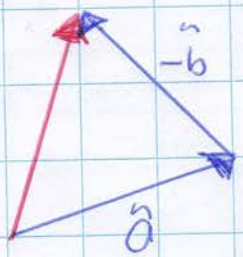
c) $-\vec{a}$

d) $-\vec{b}$

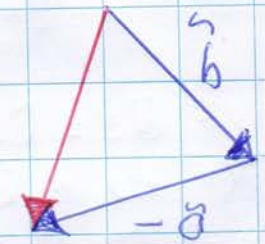
e) $\vec{a} + \vec{b}$



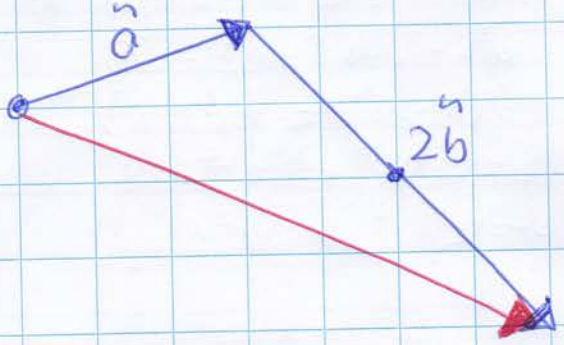
f) $\vec{a} - \vec{b}$



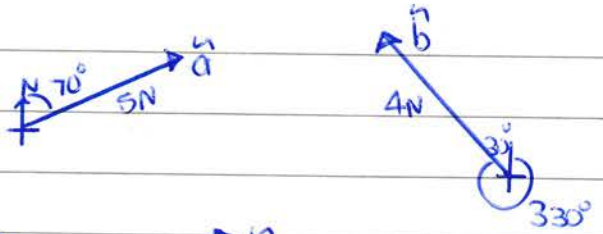
g) $\vec{b} - \vec{a}$



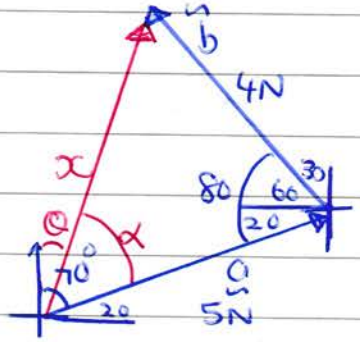
h) $\vec{a} + 2\vec{b}$



8.



a) $\vec{a}+\vec{b}$



$x^2 = 5^2 + 4^2 - 2(5)(4)\cos 80$

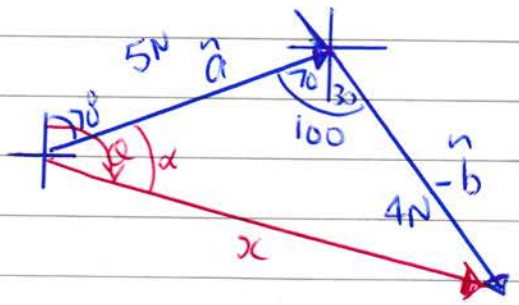
$x = 5.8\text{N}$

$\frac{\sin\alpha}{4} = \frac{\sin 80}{5.8}$

$\alpha = 42.6°$

$\therefore \theta = 70 - 42.6° = 27.54 \approx 028°$

b) $\vec{a}-\vec{b}$



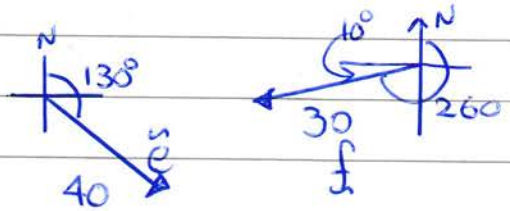
$x^2 = 5^2 + 4^2 - 2(5)(4)\cos 100$

$x = 6.9\text{ N}$

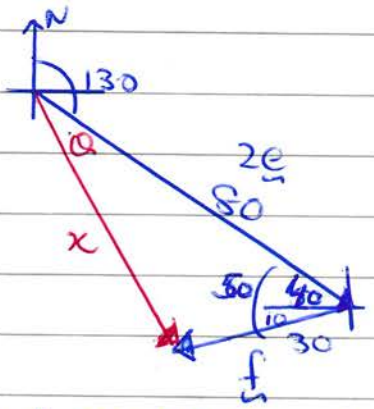
$\frac{\sin\alpha}{4} = \frac{\sin 100}{6.9}$ $\alpha = 34.67°$

$\therefore \theta = 70 + 34.67° \approx 105°$

9.



a) $2\vec{e}+\vec{f}$



$x^2 = 80^2 + 30^2 - 2(80)(30)\cos 50$

$x = 64.9 \approx 65\text{ units}$

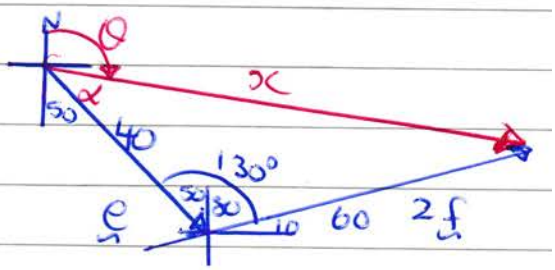
$\frac{\sin\theta}{30} = \frac{\sin 50}{64.9}$

$\theta = 20.73$

$\therefore$ bearing is $130 + 20.73$

$\approx 151°$

b) $\vec{e}-2\vec{f}$



$x^2 = 40^2 + 60^2 - 2(40)(60)\cos 130$

$x = 91\text{ units}$

$\frac{\sin\alpha}{60} = \frac{\sin 130}{91}$ $\alpha = 30.3°$

$\therefore \theta = 130 - 30.3°$

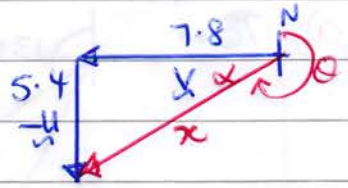
$\approx 100°$

10. $\underset{\sim}{v}$ ← 7.8 m/sec

$\underset{\sim}{u}$ ↑ 5.4 m/sec

in 5 seconds

$\underset{\sim}{a} = \dfrac{\underset{\sim}{v} - \underset{\sim}{u}}{\text{time}}$



$x^2 = 5.4^2 + 7.8^2 \qquad x = \sqrt{90}$ m/sec

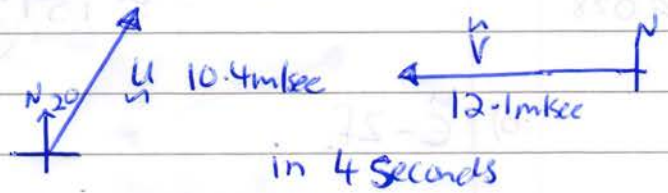
$\hat{a} = \dfrac{\sqrt{90}}{5} = 1.8974$ m/sec^2

$\simeq 1.9$ m/sec^2

$\tan\alpha = \dfrac{5.4}{7.8} \qquad \alpha = 34.69$

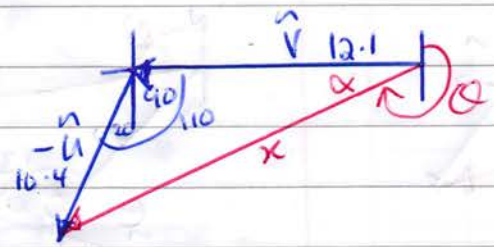
$\therefore \theta = 270 - 34.69 \simeq 235^\circ$

11.



in 4 seconds

$\underset{\sim}{a} = \dfrac{\underset{\sim}{v} - \underset{\sim}{u}}{\text{time}}$



$x^2 = 12.1^2 + 10.4^2 - 2(12.1)(10.4)\cos 110$

$x = 18.46$ m/sec

$a = \dfrac{18.46}{4} \simeq 4.6$ m/sec^2

$\dfrac{\sin\alpha}{10.4} = \dfrac{\sin 110}{18.46} \qquad \alpha = 31.97$

$\therefore \theta = 270 - 31.97 \simeq 238^\circ$

12 a) $\lambda\underset{\sim}{a} = \mu\underset{\sim}{b}$

$\lambda = 0$ & $\mu = 0$

b) $3\lambda\underset{\sim}{a} = 5\mu\underset{\sim}{b}$

$3\lambda = 0$ & $5\mu = 0$

$\lambda = 0 \qquad \mu = 0$

c) $(\lambda - 3)\underset{\sim}{a} = (\mu + 4)\underset{\sim}{b}$

$\lambda - 3 = 0 \qquad \mu + 4 = 0$

$\lambda = 3 \qquad \mu = -4$

d) $\lambda\underset{\sim}{a} - 2\underset{\sim}{a} = 5\underset{\sim}{b} - \mu\underset{\sim}{b}$

$(\lambda - 2)\underset{\sim}{a} = (5 - \mu)\underset{\sim}{b}$

$\lambda - 2 = 0$ & $5 - \mu = 0$

$\lambda = 2 \qquad \mu = 5$

e) $\lambda\underset{\sim}{a} - 2\underset{\sim}{b} = \mu\underset{\sim}{b} + 5\underset{\sim}{a}$

$\lambda\underset{\sim}{a} - 5\underset{\sim}{a} = \mu\underset{\sim}{b} + 2\underset{\sim}{b}$

$(\lambda - 5)\underset{\sim}{a} = (\mu + 2)\underset{\sim}{b}$

$\lambda - 5 = 0$ & $\mu + 2 = 0$

$\lambda = 5 \qquad \mu = -2$

f) $(\lambda + \mu - 4)\underset{\sim}{a} = (\mu - 3\lambda)\underset{\sim}{b}$

$\lambda + \mu - 4 = 0$ & $\mu - 3\lambda = 0$

$\mu = 3\lambda$

$\lambda + 3\lambda - 4 = 0$

$4\lambda - 4 = 0$

$4\lambda = 4$

$\lambda = 1$

$\therefore \mu = 3(1)$

$\mu = 3$

g) $2\vec{a} + 3\vec{b} + \mu\vec{b} = 2\vec{b} + \lambda\vec{a}$

$2\vec{a} - \lambda\vec{a} = 2\vec{b} - 3\vec{b} + \mu\vec{b}$

$(2-\lambda)\vec{a} = (2-3+\mu)\vec{b}$

$2-\lambda = 0$ $\quad -1+\mu = 0$

$\lambda = 2$ $\quad \mu = 1$

h) $\lambda\vec{a} + \mu\vec{b} + 2\lambda\vec{b} = 5\vec{a} + 4\vec{b} + \mu\vec{a}$

$\lambda\vec{a} - 5\vec{a} - \mu\vec{a} = 4\vec{b} - \mu\vec{b} - 2\lambda\vec{b}$

$(\lambda - 5 - \mu)\vec{a} = (4 - \mu - 2\lambda)\vec{b}$

$\lambda - 5 - \mu = 0$ & $4 - \mu - 2\lambda = 0$

$\lambda - \mu = 5$ $\quad -2\lambda - \mu = -4$

$$\begin{array}{r} \lambda - \mu = 5 \\ -2\lambda - \mu = -4 \\ \hline 3\lambda = 9 \end{array}$$

$\therefore \lambda = 3$

$\therefore 3 - \mu = 5$

$\mu = -2$

i) $\lambda\vec{a} - \vec{b} + \mu\vec{b} = 4\vec{a} + \mu\vec{a} - 4\lambda\vec{b}$

$\lambda\vec{a} - 4\vec{a} - \mu\vec{a} = \vec{b} - \mu\vec{b} - 4\lambda\vec{b}$

$(\lambda - 4 - \mu)\vec{a} = (1 - \mu - 4\lambda)\vec{b}$

$\lambda - 4 - \mu = 0$ & $1 - \mu - 4\lambda = 0$

$\lambda - \mu = 4$ $\quad -4\lambda - \mu = -1$

$$\begin{array}{r} \lambda - \mu = 4 \\ -4\lambda - \mu = -1 \\ \hline 5\lambda = 5 \end{array}$$

$\therefore \lambda = 1$

$\therefore 1 - \mu = 4$

$\mu = -3$

j) $2\lambda\vec{a} + 3\mu\vec{a} - \mu\vec{b} + 2\vec{b} = \lambda\vec{b} + 2\vec{a}$

$2\lambda\vec{a} + 3\mu\vec{a} - 2\vec{a} = \lambda\vec{b} + \mu\vec{b} - 2\vec{b}$

$(2\lambda + 3\mu - 2)\vec{a} = (\lambda + \mu - 2)\vec{b}$

$2\lambda + 3\mu - 2 = 0$ & $\lambda + \mu - 2 = 0$

$\lambda = 2 - \mu$

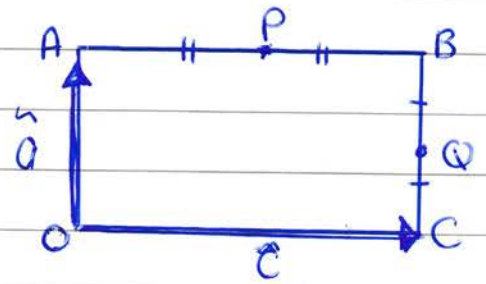
$2(2-\mu) + 3\mu - 2 = 0$

$4 - 2\mu + 3\mu - 2 = 0$

$\mu = -2$

$\therefore \lambda = 2 - (-2)$ $\quad \lambda = 4$

13.



a) $\vec{CB} = \vec{a}$ $\quad$ nak OA // CB

b) $\vec{BC} = -\vec{a}$

c) $\vec{AB} = \vec{c}$

d) $\vec{BA} = -\vec{c}$

e) $\vec{AP} = \frac{1}{2}\vec{c}$

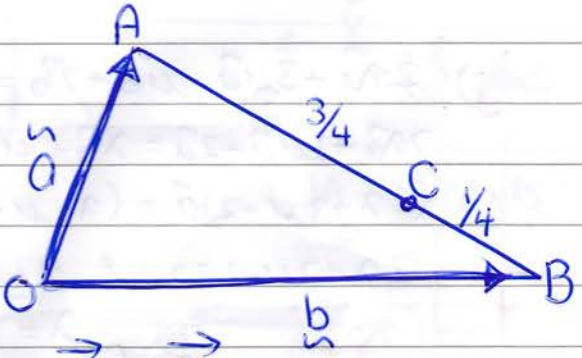
f) $\vec{OQ} = \vec{OC} + \vec{CQ}$

$= \vec{c} + \frac{1}{2}\vec{a}$

g) $\vec{OP} = \vec{OA} + \vec{AP} = \vec{a} + \frac{1}{2}\vec{c}$

h) $\vec{PQ} = \vec{PB} + \vec{BQ} = \frac{1}{2}\vec{c} - \frac{1}{2}\vec{a}$

14



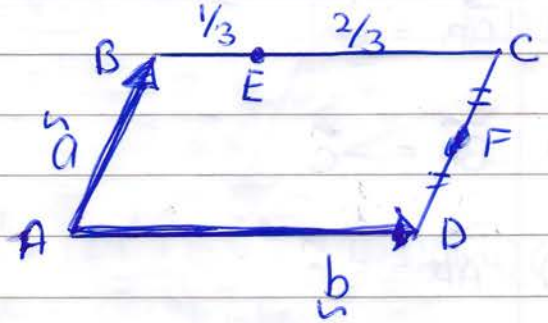
a) $\vec{AB} = \vec{AO} + \vec{OB}$
$= -\underset{\sim}{a} + \underset{\sim}{b} = \underset{\sim}{b} - \underset{\sim}{a}$

b) $\vec{AC} = \frac{3}{4}\vec{AB}$
$= \frac{3}{4}(-\underset{\sim}{a} + \underset{\sim}{b})$
$= -\frac{3}{4}\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{b} \Rightarrow \frac{3}{4}(\underset{\sim}{b} - \underset{\sim}{a})$

c) $\vec{CB} = \frac{1}{4}\vec{BA}$
$= \frac{1}{4}(-(\underset{\sim}{b} - \underset{\sim}{a}))$
$= \frac{1}{4}(\underset{\sim}{a} - \underset{\sim}{b})$

d) $\vec{OC} = \vec{OA} + \vec{AC}$
$= \underset{\sim}{a} + \frac{3}{4}\underset{\sim}{b} - \frac{3}{4}\underset{\sim}{a}$
$= \frac{1}{4}\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{b}$

15.



* BE : EC
1 : 2 ie 1/3 & 2/3

* CF : CD ⇒ CF : FD
1 : 2 1 : 1
ie midpoint

a) $\vec{AC} = \vec{AB} + \vec{BC}$
$= \underset{\sim}{a} + \underset{\sim}{b}$

b) $\vec{BE} = \frac{1}{3}\underset{\sim}{b}$

c) $\vec{DF} = \frac{1}{2}\underset{\sim}{a}$

d) $\vec{AE} = \vec{AB} + \vec{BE}$
$= \underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b}$

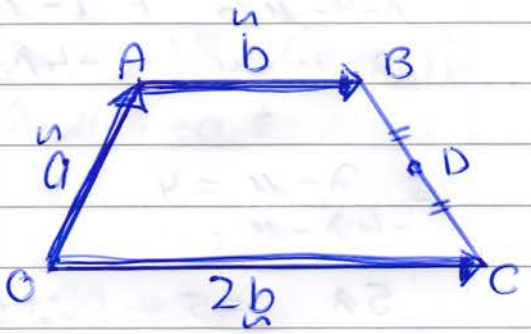
e) $\vec{AF} = \vec{AD} + \vec{DF}$
$= \underset{\sim}{b} + \frac{1}{2}\underset{\sim}{a}$

f) $\vec{BF} = \vec{BC} + \vec{CF}$
$= \underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$

g) $\vec{DE} = \vec{DC} + \vec{CE}$
$= \underset{\sim}{a} - \frac{2}{3}\underset{\sim}{b}$

h) $\vec{EF} = \vec{EC} + \vec{CF}$
$= \frac{2}{3}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$

16.



a) $\vec{OB} = \vec{OA} + \vec{AB}$
$= \underset{\sim}{a} + \underset{\sim}{b}$

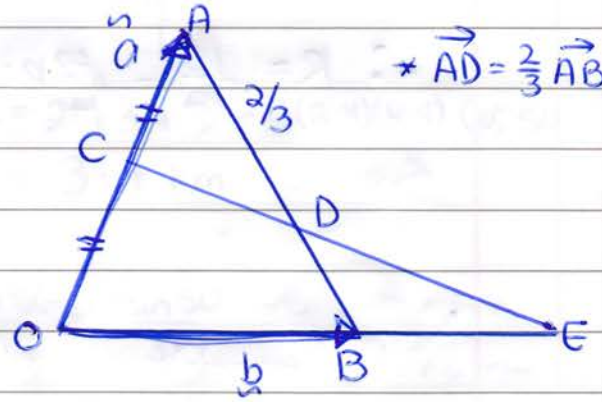
b) $\vec{OC} = 2\underset{\sim}{b}$

c) $\vec{BC} = \vec{BA} + \vec{AO} + \vec{OC}$
$= -\underset{\sim}{b} - \underset{\sim}{a} + 2\underset{\sim}{b}$
$= \underset{\sim}{b} - \underset{\sim}{a}$

d) $\vec{BD} = \frac{1}{2}\vec{BC}$
$= \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$

e) $\vec{OD} = \vec{OA} + \vec{AB} + \vec{BD}$
$= \underset{\sim}{a} + \underset{\sim}{b} + \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$
$= \underset{\sim}{a} + \underset{\sim}{b} + \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$
$= \frac{1}{2}\underset{\sim}{a} + \frac{3}{2}\underset{\sim}{b}$

17.



$* \vec{AD} = \frac{2}{3}\vec{AB}$

a) $\vec{OC} = \frac{1}{2}\underset{\sim}{a}$

b) $\vec{AB} = \vec{AO} + \vec{OB}$
$= -\underset{\sim}{a} + \underset{\sim}{b}$
$= \underset{\sim}{b} - \underset{\sim}{a}$

c) $\vec{AD} = \frac{2}{3}\vec{AB}$
$= \frac{2}{3}(\underset{\sim}{b} - \underset{\sim}{a})$

d) $\vec{CD} = \vec{CA} + \vec{AD}$
$= \frac{1}{2}\underset{\sim}{a} + \frac{2}{3}(\underset{\sim}{b} - \underset{\sim}{a})$
$= \frac{1}{2}\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b} - \frac{2}{3}\underset{\sim}{a}$
$= \frac{2}{3}\underset{\sim}{b} - \frac{1}{6}\underset{\sim}{a}$

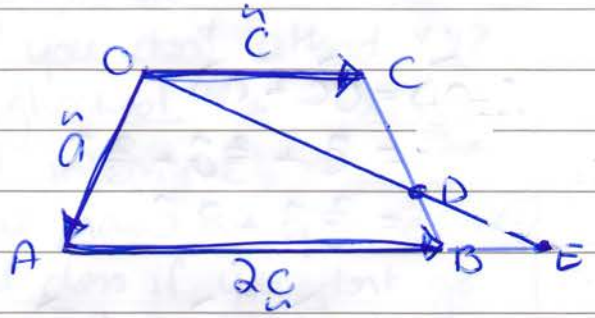
e) $\vec{CE} = h\vec{CD}$ & $\vec{OE} = k\vec{OB}$

$* \vec{OC} + \vec{CE} = \vec{OE}$
$\frac{1}{2}\underset{\sim}{a} + h(\vec{CD}) = k\vec{OB}$
$\frac{1}{2}\underset{\sim}{a} + h(\frac{2}{3}\underset{\sim}{b} - \frac{1}{6}\underset{\sim}{a}) = k\underset{\sim}{b}$
$\frac{1}{2}\underset{\sim}{a} - \frac{h}{6}\underset{\sim}{a} + \frac{2h}{3}\underset{\sim}{b} = k\underset{\sim}{b}$
$\frac{1}{2}\underset{\sim}{a} - \frac{h}{6}\underset{\sim}{a} = k\underset{\sim}{b} - \frac{2h}{3}\underset{\sim}{b}$
$\underset{\sim}{a}(\frac{1}{2} - \frac{h}{6}) = \underset{\sim}{b}(k - \frac{2h}{3})$

$\therefore \frac{1}{2} - \frac{h}{6} = 0$ & $k - \frac{2h}{3} = 0$
$\frac{1}{2} = \frac{h}{6}$
$\therefore h = 3$

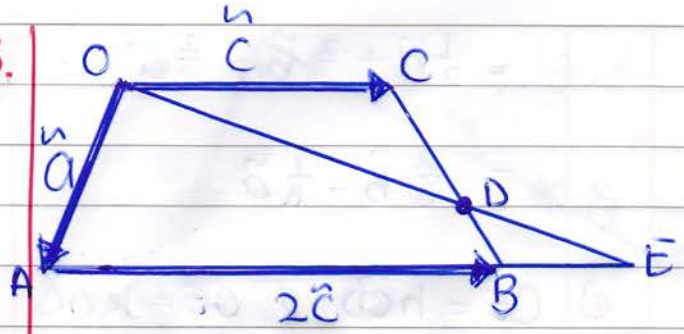
$k - \frac{2(3)}{3} = 0$ $\therefore k = 2$

18.



$* CD : DB$
$2 : 1$
ie $\frac{2}{3}$ & $\frac{1}{3}$

18.



CD : DB
2 : 1 @ 2/3 & 1/3

$\vec{OE} = h\vec{OD}$ & $\vec{AE} = k\vec{AB}$

* Find an equation to link h, k, $\vec{a}$ & $\vec{c}$

$\vec{OE} = \vec{OA} + \vec{AE}$
$h\vec{OD} = \vec{OA} + k\vec{AB}$ *

$\vec{CB} = \vec{CO} + \vec{OA} + \vec{AB}$
$= -\vec{c} + \vec{a} + 2\vec{c}$
$= \vec{a} + \vec{c}$

$\vec{CD} = \frac{2}{3}\vec{CB} = \frac{2}{3}(\vec{a} + \vec{c})$

$\therefore \vec{OD} = \vec{OC} + \vec{CD}$
$= \vec{c} + \frac{2}{3}\vec{a} + \frac{2}{3}\vec{c}$
$= \frac{2}{3}\vec{a} + \frac{5}{3}\vec{c}$

$\therefore$ * $h\vec{OD}$ $\vec{OA} + k\vec{AB}$
$h(\frac{2}{3}\vec{a} + \frac{5}{3}\vec{c}) = \vec{a} + k(2\vec{c})$
$\frac{2h}{3}\vec{a} + \frac{5h}{3}\vec{c} = \vec{a} + 2k\vec{c}$

$\frac{2h}{3}\vec{a} - \vec{a} = 2k\vec{c} - \frac{5h}{3}\vec{c}$

$\vec{a}(\frac{2h}{3} - 1) = \vec{c}(2k - \frac{5h}{3})$

$\frac{2h}{3} - 1 = 0$ & $2k - \frac{5h}{3} = 0$

$\therefore \frac{2h}{3} = 1$
$2h = 3$
$h = \frac{3}{2}$

$\therefore 2k - 5(\frac{3}{2}) = 0$
$\frac{3}{3}$

$2k - \frac{15}{2} = 0$
3

$2k - \frac{15}{6} = 0$

$2k = \frac{15}{6}$

$\therefore k = \frac{15}{12} = \frac{5}{4}$

Misc Ex 3.

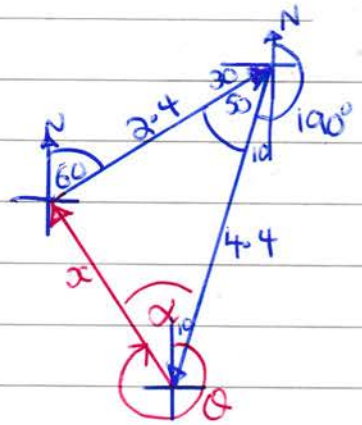
1. Six switches

on on on on on on

off off off off off off

$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64$

2.



$x^2 = 2.4^2 + 4.4^2 - 2(2.4)(4.4)\cos 50$

$x = 3.4$ km

$\frac{\sin\alpha}{2.4} = \frac{\sin 50}{3.4}$

$\alpha = 32.7°$

bearing $\Rightarrow$ $32.7 - 10 = 22.7$

$= 360 - 22.7$

$= 337°$

3. Choose 8 wines from a list of 1 dozen (12)

$^{12}C_8 = 495$

4. Same birth month

$\Rightarrow$ 12 months.

At least 3 with same month

$12 + 12 + 1$

$= 25$

5. p $\Rightarrow$ if you are in Specialist class

then

q $\Rightarrow$ you attend XYZ highschool

ie $p \rightarrow q$

Converse: $Q \rightarrow p$

If you attend XYZ highschool then you are in my specialist class

$\Rightarrow$ False, you could attend the school, but be in a different class

Contraposithe: $\bar{Q} \rightarrow \bar{p}$

If you dont attend XYZ highschool, then you are not in my class.

$\rightarrow$ true, how can you be in that class if you dont go to that school.

6. Eight people arrange for a photo.

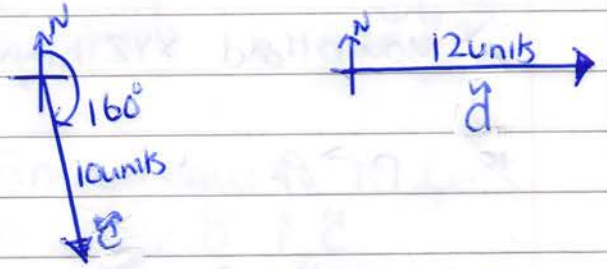
Eight are chosen from a group of 15

$^{15}C_8 \Rightarrow 6435$

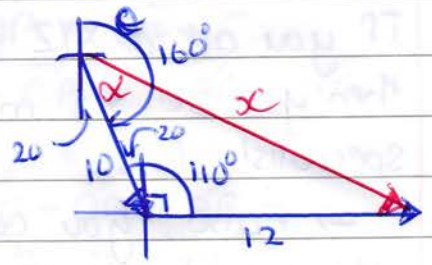
then arrange

$6435 \times 8! = 259459200$

7.



a) $\underset{\sim}{c} + \underset{\sim}{d}$



$x^2 = 10^2 + 12^2 - 2(10)(12)\cos 110°$

$x = 18.1$ units

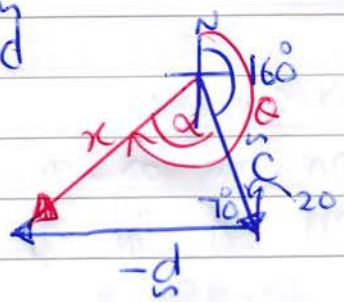
$\frac{\sin\alpha}{12} = \frac{\sin 110}{18.1}$

$\alpha = 38.6°$

Bearing $\Rightarrow 160 - 38.6$

$= 121°$

b) $\underset{\sim}{c} - \underset{\sim}{d}$



$x^2 = 10^2 + 12^2 - 2(10)(12)\cos 70$

$x = 12.7$ units

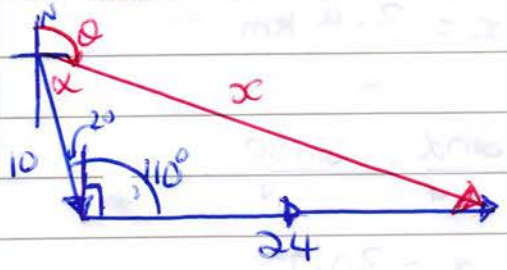
$\frac{\sin\alpha}{12} = \frac{\sin 70}{12.7}$

$\alpha = 62.4°$

$\therefore$ Bearing $= 160 + 62.4$

$= 222°$

c) $\underset{\sim}{c} + 2\underset{\sim}{d}$



$x^2 = 10^2 + 24^2 - 2(10)(24)\cos 110°$

$x = 28.98$

$= 29.0$ units

$\frac{\sin\alpha}{24} = \frac{\sin 110}{29}$

$\alpha = 51.08°$

$\therefore$ bearing $= 160 - 51.08$

$\approx 109°$

8.

a) $h\underset{\sim}{a} = k\underset{\sim}{b}$

$h=0$ & $k=0$

b) $h\underset{\sim}{a} + \underset{\sim}{b} = k\underset{\sim}{b}$

$h\underset{\sim}{a} = k\underset{\sim}{b} - \underset{\sim}{b}$

$h\underset{\sim}{a} = (k-1)\underset{\sim}{b}$

$h=0$ & $k=1$

c) $(h-3)\underset{\sim}{a} = (k+1)\underset{\sim}{b}$

$h=3$ and $k=-1$

d) $h\underset{\sim}{a} + 2\underset{\sim}{a} = k\underset{\sim}{b} - 3\underset{\sim}{a}$

$h\underset{\sim}{a} + 2\underset{\sim}{a} + 3\underset{\sim}{a} = k\underset{\sim}{b}$

$\underset{\sim}{a}(h+2+3) = k\underset{\sim}{b}$

$h=-5$ and $k=0$

e) $3h\underset{\sim}{a} + k\underset{\sim}{a} + h\underset{\sim}{b} - 2k\underset{\sim}{b} = \underset{\sim}{a} + 5\underset{\sim}{b}$

$3h\underset{\sim}{a} + k\underset{\sim}{a} - \underset{\sim}{a} = 5\underset{\sim}{b} - h\underset{\sim}{b} + 2k\underset{\sim}{b}$

$(3h+k-1)\underset{\sim}{a} = (5-h+2k)\underset{\sim}{b}$

$3h+k-1=0$ & $5-h+2k=0$

$\leftarrow h = 5+2k$

$3(5+2k)+k-1=0$

$15+6k+k-1=0$

$7k+14=0$

$k=-2$

$\therefore h = 5+2(-2)$

$h = 5-4$

$h=1$

f) $h(\underset{\sim}{a}+\underset{\sim}{b}) + k(\underset{\sim}{a}-\underset{\sim}{b}) = 3\underset{\sim}{a} + 5\underset{\sim}{b}$

$h\underset{\sim}{a} + h\underset{\sim}{b} + k\underset{\sim}{a} - k\underset{\sim}{b} = 3\underset{\sim}{a} + 5\underset{\sim}{b}$

$h\underset{\sim}{a} + k\underset{\sim}{a} - 3\underset{\sim}{a} = 5\underset{\sim}{b} - h\underset{\sim}{b} + k\underset{\sim}{b}$

$(h+k-3)\underset{\sim}{a} = (5-h+k)\underset{\sim}{b}$

$h+k-3=0$ & $5-h+k=0$

$+ \quad h+k=3$

$\quad -h+k=-5$

$\quad 2k=-2$

$\quad k=-1$

$\therefore h-1-3=0$

$h=4$

9. 10 women 6 men A group of 5

a) 3 women & 2 men

${}^{10}C_3 \times {}^{6}C_2 = 1800$

b) all women

${}^{10}C_5 \times {}^{6}C_0 = 252$

c) more women than men

${}^{10}C_3 \times {}^{6}C_2 + {}^{10}C_4 \times {}^{6}C_1 + {}^{10}C_5 \times {}^{6}C_0$

$1800 + 1260 + 252$

$= 3312$

d) more men than women

${}^{10}C_2 {}^{6}C_3 + {}^{10}C_1 \times {}^{6}C_4 + {}^{10}C_0 {}^{6}C_5$

$= 900 + 150 + 6$

$= 1056$